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Solutions of Resonant Nonlinear Schrödinger's Equation with Exotic Non-Kerr Law Nonlinearities

Anwar Ja'afar Mohamad Jawad[1,*] & Anjan Biswas[2,3,4,5]

[1]Department of Computer Technical Engineering,
Al-Rafidain University College, Baghdad-10064, Iraq

[2]Department of Mathematics and Physics,
Grambling State University, Grambling, LA 71245—2715, USA.

[3]Mathematical Modeling and Applied Computation (MMAC) Research Group,
Center of Modern Mathematical Sciences and their Applications (CMMSA)
Department of Mathematics, King Abdulaziz University, Jeddah—21589, Saudi Arabia.

[4]Department of Applied Sciences,
Cross—Border Faculty of Humanities, Economics and Engineering,
Dunare de Jos University of Galati, 111 Domneasca Street, Galati—800201, Romania.

[5]Department of Mathematics and Applied Mathematics,
Sefako Makgatho Health Sciences University, Medunsa—0204, Pretoria, South Africa.

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ABSTRACT

The solitary wave solutions of the quadratic-cubic law and Kerr-Law nonlinearity of the resonant nonlinear Schrödinger's Equation are investigated in this study. The solitary wave solutions of the resonant nonlinear Schrödinger's equations are investigated using the well-known extended simple equation method (ESEM). The field of Soliton in nonlinear fiber optics is where these equations are mainly investigated. We have obtained a new dark-bright, bell-shaped, periodic, unique, and periodic Soliton.

1. Introduction

The NLSE, or Nonlinear Schrödinger equation, is highly relevant since it may be applied to a wide range of domains. It is useful in the explanation of the propagation of light in nonlinear optical fibers, Bose-Einstein condensates, and plasmas. Furthermore, it is essential to

* Corresponding author E-mail address: anwar.jawad@ruc.edu.iq
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comprehending phenomena in quantum mechanics, mathematical biology, nano optical fibers, superconductivity, and many other domains. The widespread application of nonlinear Schrödinger type models has helped both the development of all-optical, ultra-fast switching systems and the research of long-distance optical communications. One of the inventions that had the biggest influence was the use of Soliton in optical fibers for digital information transmission [1–21].

This paper's primary objective is obtained the solitary wave solutions of the resonant nonlinear Schrödinger's equations using the well-known extended simple equation method (ESEM).

2. The Governing Resonant NLSE

Consider the resonant nonlinear Schrodinger equation, as [1]:

$$i\psi_t + \alpha\, \psi_{xx} + \gamma\, \frac{\psi\, |\psi|_{xx}}{|\psi|} + \beta\, \psi\, F(|\psi|^2) = 0 \qquad (1)$$

In Eq. (1), the function ψ represents a complex-valued function defining the profile of the complex wave. The variable x corresponds to the non-dimensional distance along the fiber, and t represents the time dimension. In Eq. (1), β and γ is the coefficient of nonlinear terms. The group velocity dispersion is denoted by the symbol α in Eq. (1). This paper will obtain soliton solutions to Eq. (1) for different types law of nonlinearities using the extended simple equation method.

3. Methodology of the Extended Simple Equation Method (ESEM)

In this section, the extended form of the Simple Equation Method (ESEM) is introduced to obtain the traveling wave solutions [1]. The nonlinear evolution equation (NLEE) can be written, as

$$H(\psi, \psi_t, \psi_x, \psi_{tt}, \psi_{xx}, \psi_{tx}, \ldots\ldots\ldots) = 0 \qquad (2)$$

Here $\psi = \psi(x,t)$ denotes a function of the variables x and t in space and time, respectively.

Step 1: Consider the following wave form $\psi(x,t)$ in Eq. (1) is a complex,

$$\psi(x,t) = u(\xi)\, e^{i\,\theta(x,t)} \qquad (3)$$

Where $\xi = k\,x + \omega\,t$, and the phase $\theta(x,t) = px + vt + \theta_0$, $u(\xi)$ is the amplitude component of the wave and ω is its speed. p is the soliton frequency; v is its wavenumber and θ_0 is the phase constant.

The conversion of Eq. (2) into the ODE is given below:

$$S(q, q', q'', q''', \ldots\ldots\ldots) = 0 \qquad (4)$$

Step 2: Considering the form of the solution for Eq. (4):

$$Q(\xi) = \sum_{j=-1}^{j=1} B_j\, f^j(\xi) \qquad (5)$$

Here, B_j is real constant.

Step 3: Find the positive integer N appeared in Eq. (5) by employing the balance rule between Eq. (4)'s non-linear terms and the highest-order derivative.

Step 4: Suppose that f satisfies the following differential equation:

$$f'(\xi) = b_0 + b_1 f(\xi) + b_2\, [f(\xi)]^2 \qquad (6)$$

where b_0, b_1, b_2 are arbitrary constants

Step 5: For different values of b_i , the solutions of Eq. (6) are given below:

When $b_0 = 0$:

$$f(\xi)=\frac{b_1\, e^{b_1(\xi+\xi_0)}}{1-b_2\, e^{b_1(\xi+\xi_0)}},\ b_1>0, \qquad (7)$$

$$f(\xi)=-\frac{b_1\, e^{b_1(\xi+\xi_0)}}{1+b_2\, e^{b_1(\xi+\xi_0)}},\ b_1<0 \qquad (8)$$

When $b_1=0$:

$$f(\xi)=\frac{\sqrt{b_0 b_2}\tan(\sqrt{b_0 b_2}\,(\xi+\xi_0))}{b_2},\ b_0 b_2>0, \qquad (9)$$

$$f(\xi)=\frac{\sqrt{-b_0 b_2}\tanh(\sqrt{-b_0 b_2}\,(\xi+\xi_0))}{b_2},\ b_0 b_2<0, \qquad (10)$$

The general solution of Eq. (6) is

$$f(\xi)=\frac{\sqrt{4b_0b_2-b_1^2}\tan\left(\frac{1}{2}\sqrt{4b_0b_2-b_1^2}\,(\xi+\xi_0)\right)-b_1}{2b_2},\ 4b_0b_2>b_1^2 \text{ and } b_2>0, \qquad (11)$$

$$f(\xi)=\frac{\sqrt{4b_0b_2-b_1^2}\tan\left(\frac{1}{2}\sqrt{4b_0b_2-b_1^2}\,(\xi+\xi_0)\right)+b_1}{2b_2},\ 4b_0b_2>b_1^2 \text{ and } b_2<0, \qquad (12)$$

Step 6: Inserting Eq. (5) with Eq. (6) in Eq. (4) and equating the coefficients of powers of f^j to zero, the result is a system of equations. The set of equations is solved and the value of constant parameters have been obtained. By carrying these constant values and the $f(\xi)$ values in Eq. (5), the solution of Eq. (2) is achieved.

4. Travelling wave solution

From Eq. (3), the following equations can be obtained:

$$i\,\psi_t=e^{i\theta}[i\,\omega\, u'-v\,\mathrm{u}] \qquad (13)$$

$$\psi_x=e^{i\,\theta}[k\,u'+i\,p\,u] \qquad (14)$$

$$\psi_{xx}=e^{i\,\theta}[k^2u''+2ip\,u'-p^2u\ ] \qquad (15)$$

Equation (1) can be decomposing into real and imaginary parts yields a pair of relations. The real and imaginary parts of Eq. (1) respectively are:

$$\beta u\,F(|u|^2)+\ k^2(\alpha+\gamma)u''-[p^2(\alpha+\gamma)+v]\,u\ =0 \qquad (16)$$

$$\omega=-2(\alpha+\gamma)p \qquad (17)$$

Application of Extended Simple Equation Method (ESEM)

In this section the Extended Simple Equation Method (ESEM) is applied to solve different types of nonlinearities:

4.1 Kerr law nonlinearity

For the Kerr law nonlinearity, $\mathrm{F(s)=s}$. In this case, Eq. (16) simplifies to

$$\beta\,u^3+\ k^2(\alpha+\gamma)u''-[p^2(\alpha+\gamma)+v]\,u\ =0 \qquad (18)$$

In order to find the values of N, apply the homogeneous balance principle to the Eq. (18). By balancing u $''$ and u^3, N+2= 3N, then N = 1. Thus $u(\xi)$ has the form that is given below:

$$u(\xi)=\ \frac{B_{-1}}{f(\xi)}+B_0+B_1f(\xi) \quad B_1\neq 0 \qquad (19)$$

$$u'(\xi)=\left[-\frac{b_0\,B_{-1}}{f^2}-\frac{b_1\,B_{-1}}{f}-b_2B_{-1}+b_0B_1+b_1B_1f+b_2B_1\,f^2\right] \qquad (20)$$

$$u''(\xi)=\left[\frac{2\,{b_0}^2\,B_{-1}}{f^3}+\frac{3b_0b_1\,B_{-1}}{f^2}+({b_1}^2+2\,b_0b_2)\frac{B_{-1}}{f}+(b_1\,b_2B_{-1}+b_0b_1B_1)+B_1(2b_0b_2+{b_1}^2)f+3\,b_1b_2B_1\,f^2+2{b_2}^2B_1\,f^3\right] \qquad (21)$$

By using Eq. (19) and Eq. (6) in Eq. (18) to get the following equation:

$$\beta\ \left[\frac{B^3{}_{-1}}{f^3}+3B_0\frac{B^2{}_{-1}}{f^2}+3B_1\frac{B^2{}_{-1}}{f}+3\,{B_0}^2\frac{B_{-1}}{f}+6B_0B_1B_{-1}+{B_0}^3+(B_{-1}B_1\ +{B_0}^2)3B_1f+3B_0{B_1}^2f^2+\right.$$

$B_1{}^3 f^3\Big] + k^2(\alpha+\gamma)\Big[\frac{2\,b_0{}^2\,B_{-1}}{f^3} + \frac{3b_0b_1\,B_{-1}}{f^2} + \left(b_1{}^2 + 2\,b_0b_2\right)\frac{B_{-1}}{f} + (b_1\,b_2B_{-1} + b_0b_1B_1) + B_1\left(2b_0b_2 + b_1{}^2\right)f + 3\,b_1b_2B_1\,f^2 + 2b_2{}^2B_1\,f^3\Big] - [p^2(\alpha+\gamma)+v]\,\Big[\frac{B_{-1}}{f} + B_0 + B_1 f\Big] = 0$ (22)

Set of Equations is obtained for different orders of f^j, $j = -3,-2,-1,0,1,2,3$

$\{\beta\, B^2{}_{-1} + k^2(\alpha+\gamma)2\,b_0{}^2\}\,\frac{B_{-1}}{f^3} = 0$

$\{\beta\,[3B_0B_{-1}] + k^2(\alpha+\gamma)3b_0b_1\}\,\frac{B_{-1}}{f^2} = 0$

$\{\beta\,[3B_1B_{-1} + 3\,B_0{}^2] + k^2(\alpha+\gamma)[(b_1{}^2 + 2\,b_0b_2)] - [p^2(\alpha+\gamma) + v]\}\,\frac{B_{-1}}{f} = 0$

$\beta\,[6B_0B_1B_{-1} + B_0{}^3] + k^2(\alpha+\gamma)[(b_1\,b_2B_{-1} + b_0b_1B_1)] - [p^2(\alpha+\gamma)+v]\,B_0 = 0$

$\{3\beta\,(B_{-1}B_1 + B_0{}^2) + k^2(\alpha+\gamma)(2b_0b_2 + b_1{}^2) - [p^2(\alpha+\gamma)+v]\}\,B_1 f = 0$

$3\{\beta\,B_0B_1 + k^2(\alpha+\gamma)\,b_1b_2\}\,B_1\,f^2 = 0$

$\{\beta\,B_1{}^2 + k^2(\alpha+\gamma)2b_2{}^2\}\,B_1{}^3 f^3 = 0$ (23)

Constant values of $B_{-1}, B_0, B_1, b_0, b_1, b_2$, are obtained for the following cases:

Case I:

$b_0 = 0$

Then :

$B_{-1} = 0,\ B_0 = 0,\ B_1 = \mp ikb_2\sqrt{\frac{2(\alpha+\gamma)}{\beta}}$,

$v = \left(k^2b_1{}^2 - p^2\right)(\alpha+\gamma)$

$\psi_1(\xi) = \mp ikb_2\sqrt{\frac{2(\alpha+\gamma)}{\beta}}\frac{b_1\,e^{b_1(\xi+\xi_0)}}{1-b_2\,e^{b_1(\xi+\xi_0)}}e^{i\,\theta}$, $b_1 > 0$, (24)

$\psi_2(\xi) = \mp ikb_2\sqrt{\frac{2(\alpha+\gamma)}{\beta}}\left(-\frac{b_1\,e^{b_1(\xi+\xi_0)}}{1+b_2\,e^{b_1(\xi+\xi_0)}}\right)e^{i\,\theta}$, $b_1 < 0$ (25)

where: $\xi = k\,x - 2(\alpha+\gamma)p\,t$, and the phase $\theta = px + \left(k^2b_1{}^2 - p^2\right)(\alpha+\gamma)\mathrm{t}$ (26)

Case II:

$b_1 = 0$

Family I:

$B_{-1} = \mp ib_0k\sqrt{\frac{2\,(\alpha+\gamma)}{\beta}}$, $B_0 = 0$,

$B_1 = \mp i\,b_2k\sqrt{\frac{2(\alpha+\gamma)}{\beta}}$, $v = [4k^2b_0b_2 - p^2](\alpha+\gamma)$

$\psi_3(\xi) = \mp ik\sqrt{\frac{2\,(\alpha+\gamma)}{\beta}}\left[b_0\cot\left(\sqrt{b_0b_2}\,(\xi+\xi_0)\right) + b_2\tan\left(\sqrt{b_0b_2}\,(\xi+\xi_0)\right)\right]e^{i\,\theta}$, $b_0b_2 > 0$, (27)

$\psi_4(\xi) = \mp ik\sqrt{\frac{2\,(\alpha+\gamma)}{\beta}}\left[b_0\coth\left(\sqrt{b_0b_2}\,(\xi+\xi_0)\right) + b_2\tanh\left(\sqrt{b_0b_2}\,(\xi+\xi_0)\right)\right]e^{i\,\theta}$, $b_0b_2 < 0$ (28)

where: $\xi = k\,x - 2(\alpha+\gamma)p\,t$, and the phase $\theta = px + [4k^2b_0b_2 - p^2](\alpha+\gamma)t$ (29)

Family II:

$B_{-1} = \mp ib_0k\sqrt{\frac{2\,(\alpha+\gamma)}{\beta}}$, $B_0 = 0$, ,

$B_1 = \mp i\,k\,b_2\sqrt{\frac{2(\alpha+\gamma)}{\beta}}$,

$v = -[8\,k^2b_0b_2 + p^2](\alpha+\gamma)$

then

$\psi_5(\xi) = \mp ik\sqrt{\frac{2\,(\alpha+\gamma)}{\beta}}\left[b_0\cot\left(\sqrt{b_0b_2}\,(\xi+\xi_0)\right) + b_2\tan\left(\sqrt{b_0b_2}\,(\xi+\xi_0)\right)\right]e^{i\,\theta}$, $b_0b_2 > 0$, (30)

$\psi_6(\xi) = \mp ik\sqrt{\frac{2\,(\alpha+\gamma)}{\beta}}\left[b_0\coth\left(\sqrt{b_0b_2}\,(\xi+\xi_0)\right) + b_2\tanh\left(\sqrt{b_0b_2}\,(\xi+\xi_0)\right)\right]e^{i\,\theta}$, $b_0b_2 < 0$ (31)

where: $\xi = k\,x - 2(\alpha+\gamma)p\,t$, and the phase $\theta = px - [8\,k^2b_0b_2 + p^2](\alpha+\gamma)t$ (32)

Case III

$B_{-1}=0, B_0=\mp i\, kb_1\sqrt{\frac{(\alpha+\gamma)}{2\beta}}$,

$B_1=\mp i\, b_2\, k\sqrt{\frac{2(\alpha+\gamma)}{\beta}}$,

$v=-\frac{3kb_1}{\sqrt{2}}\sqrt{\beta(\alpha+\gamma)}+(\alpha+\gamma)\left\{k^2\left(2b_0b_2+{b_1}^2\right)-p^2\right\}$

Then:

$$\psi_7(\xi)=\mp i\, k\sqrt{\frac{(\alpha+\gamma)}{2\beta}}\,[b_1+2\, b_2\, f(\xi)]\ e^{i\,\theta} \quad (33)$$

Where:

$$f(\xi)=\frac{\sqrt{4b_0b_2-b_1^2}\tan\left(\frac{1}{2}\sqrt{4b_0b_2-b_1^2}\,(\xi+\xi_0)\right)-b_1}{2b_2}$$

, $4b_0b_2>b_1^2$ and $b_2>0$, (34)

$$f(\xi)=\frac{\sqrt{4b_0b_2-b_1^2}\tan\left(\frac{1}{2}\sqrt{4b_0b_2-b_1^2}\,(\xi+\xi_0)\right)+b_1}{2b_2}$$

, $4b_0b_2>b_1^2$ and $b_2<0$, (35)

$\xi=\ k\,x-2(\alpha+\gamma)p\,t$, and the phase $\theta=px+v\,t$, $v=-\frac{3kb_1}{\sqrt{2}}\sqrt{\beta(\alpha+\gamma)}+(\alpha+\gamma)\left\{k^2\left(2b_0b_2+{b_1}^2\right)-p^2\right\}$ (36)

Quadratic-cubic law

The general form can be written as $F(s)=c_1\sqrt{s}+c_2\,s$. In this case, Eq. (16) simplifies to:

$$\beta\,[c_1u^2+c_2\,u^3\,]+k^2(\alpha+\gamma)u''-[p^2(\alpha+\gamma)+v]\,u\ =0 \quad (37)$$

In order to find the values of N, apply the homogeneous balance principle to the Eq. (37). By balancing u $''$ and u^3 , N+2= 3N, then N = 1. Thus Eq. (5) has the form that is given below:

$$u(\xi)=\frac{B_{-1}}{f(\xi)}+B_0+B_1f(\xi)\quad B_1\neq 0 \quad (38)$$

By using Eq. (38) and Eq. (6) in Eq. (37) to get the following equation:

$$\beta c_1\left[\frac{{B^2}_{-1}}{f^2}+2B_0\frac{B_{-1}}{f}+2B_{-1}B_1+{B_0}^2+2B_0\,B_1f+\ {B_1}^2f^2\right]+\beta\,c_2\left[\frac{{B^3}_{-1}}{f^3}+3B_0\frac{{B^2}_{-1}}{f^2}+3B_1\frac{{B^2}_{-1}}{f}+3\,{B_0}^2\frac{B_{-1}}{f}+6B_0B_1B_{-1}+{B_0}^3+\left(B_{-1}B_1\ +{B_0}^2\right)3B_1f+3B_0{B_1}^2f^2+{B_1}^3f^3\right]+k^2(\alpha+\gamma)\left[\frac{2\,{b_0}^2\,B_{-1}}{f^3}+\frac{3b_0b_1\,B_{-1}}{f^2}+\left({b_1}^2+2\,b_0b_2\right)\frac{B_{-1}}{f}+(b_1\,b_2B_{-1}+b_0b_1B_1)+B_1\left(2b_0b_2+{b_1}^2\right)f+3\,b_1b_2B_1\,f^2+2{b_2}^2B_1\,f^3\right]-[p^2(\alpha+\gamma)+v]\ \left[\frac{B_{-1}}{f}+B_0+B_1f\right]\ =0 \quad (39)$$

Set of Equations is obtained for different orders of f^j , $j=-3,-2,-1,0,1,2,3$

$$\left\{\beta\,c_2{B^2}_{-1}+2\,{b_0}^2\,k^2(\alpha+\gamma)\right\}B_{-1}=0$$
$$\{\beta c_1B_{-1}+3\beta\,c_2\,B_0B_{-1}+\ k^2(\alpha+\gamma)3b_0b_1\}B_{-1}=0$$
$$\left\{2\beta c_1B_0+\beta\,c_2\left[3B_1B_{-1}+3\,{B_0}^2\right]+\ k^2(\alpha+\gamma)\left({b_1}^2+2\,b_0b_2\right)-[p^2(\alpha+\gamma)+v]\right\}B_{-1}=0$$
$$\beta c_1\left[2B_{-1}B_1+{B_0}^2\right]+\beta\,c_2\left[6B_0B_1B_{-1}+{B_0}^3\right]+\ k^2(\alpha+\gamma)(b_1\,b_2B_{-1}+b_0b_1B_1)-[p^2(\alpha+\gamma)+v]\,B_0\ =0$$
$$\left\{\beta c_1[2B_0\,]+\beta\,c_2\left[\left(B_{-1}B_1\ +{B_0}^2\right)3\right]+\ k^2(\alpha+\gamma)\left[\left(2b_0b_2+{b_1}^2\right)\right]-[p^2(\alpha+\gamma)+v]\right\}B_1=0$$
$$\{\beta c_1B_1+3\,\beta\,c_2B_0B_1+\ k^2(\alpha+\gamma)3\,b_1b_2\ \}\ B_1=0$$
$$\left\{\beta\,c_2\,{B_1}^2+\ k^2(\alpha+\gamma)2{b_2}^2\right\}B_1=0 \quad (40)$$

Constant values of $B_{-1}, B_0, B_1, b_0, b_1, b_2$, are obtained for the following cases:

Case I:

$b_0=0$

Then :

$B_{-1}=0,\ \ B_0=0,\ \ B_1=\mp ikb_2\sqrt{\frac{2(\alpha+\gamma)}{\beta}}$,

$v=\left(k^2{b_1}^2-p^2\right)(\alpha+\gamma)$,

$$b_1 = -\,i\,\frac{c_1}{3k}\sqrt{\frac{2\beta}{(\alpha+\gamma)}} \quad (41)$$

$$\psi_8(\xi) = \mp ikb_2\sqrt{\frac{2(\alpha+\gamma)}{\beta}}\;\frac{b_1\,e^{b_1(\xi+\xi_0)}}{1-b_2\,e^{b_1(\xi+\xi_0)}}e^{i\,\theta}\;,\; b_1 > 0, \quad (42)$$

$$\psi_9(\xi) = \pm\, ikb_2\sqrt{\frac{2(\alpha+\gamma)}{\beta}}\left(\frac{b_1\,e^{b_1(\xi+\xi_0)}}{1+b_2\,e^{b_1(\xi+\xi_0)}}\right)e^{i\,\theta}\;,\; b_1 < 0 \quad (43)$$

where: $\xi = k\,x - 2(\alpha+\gamma)p\,t$, and the phase $\theta = px + v$ t.

Case II:

$b_1 = 0 \quad B_{-1} = \mp ib_0k\sqrt{\frac{2\,(\alpha+\gamma)}{c_2\beta}}$,

$B_0 = -\frac{c_1}{3\,c_2}$, $B_1 = \mp i\,k\,b_2\sqrt{\frac{2(\alpha+\gamma)}{\beta}}$,

$$v = (1 + 3\sqrt{c_2})\,2\,k^2\,(\alpha+\gamma)b_0b_2 - \beta\frac{{c_1}^2}{3\,c_2} - p^2(\alpha+\gamma) \quad (44)$$

Then:

$$\psi_{10}(\xi) = \left[\mp ik\sqrt{\frac{2(\alpha+\gamma)}{\beta}}\left\{b_0\cot\left(\sqrt{b_0b_2}\,(\xi+\xi_0)\right) + b_2\tan\left(\sqrt{b_0b_2}\,(\xi+\xi_0)\right)\right\} - \frac{c_1}{3\,c_2}\right]e^{i\,\theta}\;,\; b_0b_2 > 0, \quad (45)$$

$$\psi_{11}(\xi) = \left[\mp ik\sqrt{\frac{2(\alpha+\gamma)}{\beta}}\left\{b_0\coth\left(\sqrt{b_0b_2}\,(\xi+\xi_0)\right) + b_2\tanh\left(\sqrt{b_0b_2}\,(\xi+\xi_0)\right)\right\} - \frac{c_1}{3\,c_2}\right]e^{i\,\theta}\;,\; b_0b_2 < 0, \quad (46)$$

where: $\xi = k\,x - 2(\alpha+\gamma)p\,t$, and the phase $\theta = px + v\,t$,

Case III

$B_{-1} = 0, B_0 = \mp i\,kb_1\sqrt{\frac{(\alpha+\gamma)}{2c_2\beta}} - \frac{c_1}{3c_2}$,

$$B_1 = \mp i\,b_2\,k\sqrt{\frac{2(\alpha+\gamma)}{c_2\beta}}\;,\quad v = \beta\,(2c_1 + 3\,c_2\,B_0)B_0 + (\alpha+\gamma)\left\{k^2\left(2b_0b_2 + {b_1}^2\right) - p^2\right\} \quad (47)$$

Then:

$$\psi_{12}(\xi) = \left[\mp i\,kb_1\sqrt{\frac{(\alpha+\gamma)}{2c_2\beta}} - \frac{c_1}{3c_2} \mp i\,b_2\,k\sqrt{\frac{2(\alpha+\gamma)}{c_2\beta}}f(\xi)\right]e^{i\,\theta} \quad (48)$$

Where:

$$(\xi) = \frac{\sqrt{4b_0b_2-b_1^2}\tan\left(\frac{1}{2}\sqrt{4b_0b_2-b_1^2}\,(\xi+\xi_0)\right)-b_1}{2b_2}\;,\; 4b_0b_2 > b_1^2 \text{ and } b_2 > 0\,, \quad (49)$$

$$f(\xi) = \frac{\sqrt{4b_0b_2-b_1^2}\tan\left(\frac{1}{2}\sqrt{4b_0b_2-b_1^2}\,(\xi+\xi_0)\right)+b_1}{2b_2}\;,\; 4b_0b_2 > b_1^2 \text{ and } b_2 < 0\,, \quad (50)$$

$\xi = k\,x - 2(\alpha+\gamma)p\,t$, and the phase $\theta = px + v\,t$,

5. Conclusion

Using the expanded version of simple equation method, new exact solitary wave solutions for resonant nonlinear Schrödinger's equation are effectively generated in this study. Certain solutions are shown graphically by assigning specific values to the arbitrary parameters and arbitrary constants. Periodic bell-shaped, dark-bright, unique, and periodic Solitons are recovered. These answers serve as a roadmap for comprehending nonlinear physical processes. The computational work confirms the suggested method's ease of use and simplicity. The various models that arise in mathematics and physics can likewise be used with this methodology.

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Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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